# Paper Boats
# Mechanical Engineering

**Objective**
- This lesson teaches students about the physics of buoyancy. The activity should be used to expand what they have learned in class.

**Standards and Objectives**
- $8^{th}$ Grade Standard 4, Objective 3
- Physics Standard 2, Objective 3

**Learning Outcomes**
Students will learn:
- About forces involved in buoyancy
- Effective designs for buoyant objects
- How to use the scientific method
- How to construct a buoyant object that can hold the weight of two team members
- That engineers work in teams

**Essential Questions**
- How do things float?
- Are science and math involved with designing boats?
- How can the best boat be constructed from butcher paper and duct tape?

**Time Required (Itemized)**
- Introduction of the activity: logistics, research, etc. (20 minutes)
- Work on the activity: determine if class time will be used or if students will work at home (approximately 60 minutes of class time)
- Testing the boats in the swimming pool (1 class period)

**Assessments**
- Students will be graded on their boat design: Does it float? How long does it float? Did students base their design on research?
- Students could also submit a 1-page paper to the teacher explaining the design and rationale for the design.
- Students could present their boats to the class and talk about why they chose their design.

**Materials**
- 6 yards of Butcher paper
- 1 roll of Duct tape
- 1 stopwatch or timing device for each group

- Swimming pool

**Lesson Description**

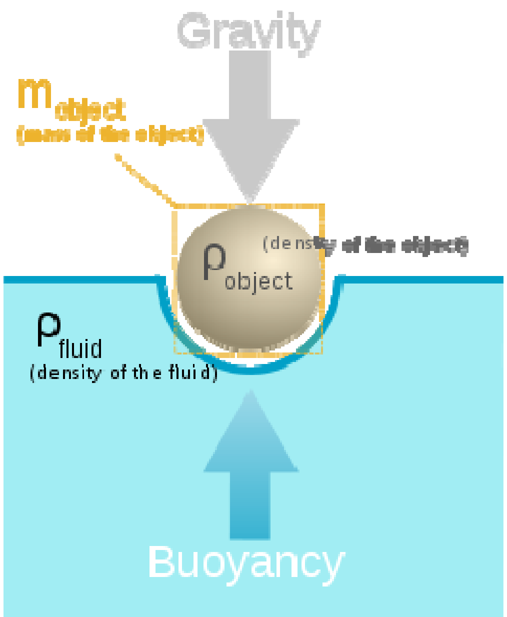
- Students are introduced to principles of buoyancy.
- Students are presented with the design competition:  who can build a boat out of paper and duct tape that will float for the longest?
- Students should work in teams of 3 to construct a boat.
- Each team should take pictures of their boat—to be used later in the lesson as a discussion item.
- Have the class look at each teams' boats and hypothesize which will float the best.
- During the boat testing, each team must have 2 members in the boat and one on the side.
- The team member on the side will use the stopwatch to measure the length of time before the boat completely sinks in the water.
- After the competition is over, display the boat designs to the students and talk about what worked and what did not.
- Decide as a class which design is the best, and talk about modifications that would make it more effective.

**Note from the College of Engineering**

- Please be sure to take the proper precautions when planning this activity, ex. parent consent forms, life jackets (if necessary), etc.

# Buoyancy

From Wikipedia (www.en.wikipedia.com)




The forces at work in buoyancy

In physics, **buoyancy** is the upward force that keeps things afloat. The net upward buoyancy force is equal to the magnitude of the weight of fluid displaced by the body. This force enables the object to float or at least seem lighter.

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## Archimedes' principle

It is named after Archimedes of Syracuse, who first discovered this law. According to Archimedes' principle, "Any object, wholly or partly immersed in a fluid, is buoyed up by a force equal to the weight of the fluid displaced by the object."

Archimedes' principle does not consider the surface tension (capillarity) acting on the body.

The weight of the displaced fluid is directly proportional to the volume of the displaced fluid (if the surrounding fluid is of uniform density). Thus, among completely submerged objects with equal masses, objects with greater volume have greater buoyancy.

Suppose a rock's weight is measured as 10 newtons when suspended by a string in a vacuum. Suppose that when the rock is lowered by the string into water, it displaces water of weight 3 newtons. The force it then exerts on the string from which it hangs would be 10 newtons minus the 3 newtons of buoyant force: 10 − 3 = 7 newtons. Buoyancy reduces the apparent weight of objects that have sunk completely to the sea floor. It is generally easier to lift an object up through the water than it is to pull it out of the water.

The density of the immersed object relative to the density of the fluid can easily be calculated without measuring any volumes:

$$\frac{\text{density of object}}{\text{density of fluid}} = \frac{\text{weight}}{\text{weight} - \text{apparent immersed weight}}$$

# Forces and equilibrium

This is the equation to calculate the pressure inside a fluid in equilibrium. The corresponding equilibrium equation is:

$$\mathbf{f} + \operatorname{div}\sigma = 0$$

where $\mathbf{f}$ is the force density exerted by some outer field on the fluid, and $\sigma$ is the stress tensor. We know that in our case the stress tensor is proportional to the identity tensor: $\sigma_{ij} = -p\delta_{ij}$. Here $\delta_{ij}$ is the Kronecker delta symbol. Using this the above equation becomes:

$$\mathbf{f} = \nabla p$$

Now let's assume that the outer force field is conservative, that is it can be written as the negative gradient of some scalar valued function: :$\mathbf{f} = -\nabla\Phi$. Hence we have:

$$\nabla(p + \Phi) = 0 \Longrightarrow p + \Phi = \text{constant}.$$

As we see, we got that the shape of the open surface of a fluid equals the equipotential plane of the applied outer conservative force field. Now let's put the z axis pointing downwards. In our case we have gravity, so $\Phi = -\rho g z$ where g is the gravitational acceleration, $\rho$ is the mass density of the fluid. Let the constant be zero, that is the pressure zero where z is zero. So the pressure inside the fluid, when it is subject to gravity:

$$p = \rho g z$$

So as we see, pressure increases with depth below the surface of a liquid, as z denotes the distance from the surface of the liquid into it. Any object with a non-zero vertical depth will have different pressures on its top and bottom, with the pressure on the bottom being greater. This difference in pressure causes the upward buoyancy forces.

The buoyant force exerted on a body can now be calculated easily, since we know the internal pressure of the fluid. We know that the force exerted on the body can be calculated by integrating the stress tensor over the surface of the body:

$$\mathbf{F} = \oint \sigma \, d\mathbf{A}$$

The surface integral can be transformed into a volume integral with the help of the Gauss-Ostrogradsky theorem :

$$\mathbf{F} = \int \mathrm{div}\sigma \, dV = -\int \mathbf{f} \, dV = -\rho \mathbf{g} \int \, dV = -\rho \mathbf{g} V$$

where *V* is obviously the measure of the volume in contact with the fluid, that is the volume of the submerged part of the body. Since the fluid doesn't exert force on the part of the body which is outside of it.

The magnitude of buoyant force may be appreciated a bit more from the following argument. Consider any object of arbitrary shape and volume *V* surrounded by a liquid. The force the liquid exerts on an object within the liquid is equal to the weight of the liquid with a volume equal to that of the object. This force is applied in a direction opposite to gravitational force that is, of magnitude:

$$\rho V_{\mathrm{disp}} g,$$

where $\rho$ is the density of the liquid, $V_{\mathrm{disp}}$ is the volume of the displaced body of liquid , and $g$ is the gravitational acceleration at the location in question.

Now, if we replace this volume of liquid by a solid body of the exact same shape, the force the liquid exerts on it must be exactly the same as above. In other words the "buoyant force" on a submerged body is directed in the opposite direction to gravity and is equal in magnitude to :

$$\rho V g$$

The net force on the object is thus the sum of the buoyant force and the object's weight

$$F_{\mathrm{net}} = mg - \rho V g$$

If the buoyancy of an (unrestrained and unpowered) object exceeds its weight, it tends to rise. An object whose weight exceeds its buoyancy tends to sink.

Commonly, the object in question is floating in equilibrium and the sum of the forces on the object is zero, therefore;

$$mg = \rho V g$$

and therefore;

$$m = \rho V$$

showing that the depth to which a floating object will sink (its "**buoyancy**") is independent of the variation of the gravitational acceleration at various locations on the surface of the Earth.

> (*Note: If the liquid in question is seawater, it will not have the same density (* $\rho$*) at every location. For this reason, a ship may display a Plimsoll line.)*

It is common to define a *buoyant mass* $m_b$ that represents the effective mass of the object with respect to gravity

$$m_b = m_{\mathrm{o}} \cdot \left(1 - \frac{\rho_{\mathrm{f}}}{\rho_{\mathrm{o}}}\right)$$

where $m_{\mathrm{o}}$is the true (vacuum) mass of the object, whereas $\rho_o$ and $\rho_f$ are the average densities of the object and the surrounding fluid, respectively. Thus, if the two densities are equal, $\rho_o = \rho_f$, the object appears to be weightless. If the fluid density is greater than the average density of the object, the object floats; if less, the object sinks.

# Compressive fluids

The atmosphere's density depends upon altitude. As an airship rises in the atmosphere, its buoyancy decreases as the density of the surrounding air decreases. As a submarine expels water from its buoyancy tanks (by pumping them full of air) it rises because its volume is constant (the volume of water it displaces if it is fully submerged) as its weight is decreased.

### Compressible objects

As a floating object rises or falls, the forces external to it change and, as all objects are compressible to some extent or another, so does the object's volume. Buoyancy depends on volume and so an object's buoyancy reduces if it is compressed and increases if it expands.

If an object at equilibrium has a compressibility less than that of the surrounding fluid, the object's equilibrium is stable and it remains at rest. If, however, its compressibility is greater, its equilibrium is then unstable, and it rises and expands on the slightest upward perturbation, or falls and compresses on the slightest downward perturbation.

Submarines rise and dive by filling large tanks with seawater. To dive, the tanks are opened to allow air to exhaust out the top of the tanks, while the water flows in from the bottom. Once the weight has been balanced so the overall density of the submarine is equal to the water around it, it has neutral buoyancy and will remain at that depth. Normally, precautions are taken to ensure that no air has been left in the tanks. If air were left in the tanks and the submarine were to descend even slightly, the increased pressure of the water would compress the remaining air in the tanks, reducing its volume. Since buoyancy is a function of volume, this would cause a decrease in buoyancy, and the submarine would continue to descend.

The height of a balloon tends to be stable. As a balloon rises it tends to increase in volume with reducing atmospheric pressure, but the balloon's cargo does not expand. The average density of the balloon decreases less, therefore, than that of the surrounding air. The balloon's buoyancy decreases because the weight of the displaced air is reduced. A rising balloon tends to stop rising. Similarly, a sinking balloon tends to stop sinking.

## Density

If the weight of an object is less than the weight of the displaced fluid when fully submerged, then the object has an average density that is less than the fluid and has a buoyancy that is greater than its own weight. If the fluid has a surface, such as water in a lake or the sea, the object will float at a level where it displaces the same weight of fluid as the weight of the object. If the object is immersed in the fluid, such as a submerged submarine or air in a balloon, it will tend to rise. If the object has exactly the same density as the fluid, then its buoyancy equals its weight. It will remain submerged in the fluid, but it will neither sink nor float. An object with a higher average density than the fluid has less buoyancy than weight and it will sink. A ship floats because although it is made of steel, which is much denser than water, it encloses a volume of air which is lighter than water, and the resulting shape has an average density less than that of the water.